Temperature-independent Casimir–Polder forces in arbitrary geometries

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We show that the Casimir–Polder potential of a particle in an energy eigenstate at nonretarded distance from a well-conducting body of arbitrary shape is independent of the environment temperature. This is true even when the thermal photon numbers at the relevant atomic transition energies are large. A compact expression is obtained for the temperature-independent potential, which can greatly simplify calculations in nontrivial geometries for experimentally relevant systems such as Rydberg atoms and polar molecules. We give criteria for the validity of our temperature-independent result. They are illustrated by numerical studies of a particle near a gold sphere or inside a gold cylindrical cavity.

Introduction. Theoretical and experimental research into the Casimir and Casimir–Polder (CP) effects [1] have seen a phenomenal surge of activity over the last good decade [2]. These effects, together with the van der Waals forces often collectively termed dispersion forces, are consequences of ever present fluctuations, quantum and thermal, of charges, currents and electromagnetic fields.

Motivated by experimental progress and an increasing number of technological applications, one has arrived at a number of very general insights into the behaviour of dispersion forces in recent years. It was shown that the Casimir interaction between bodies that are mirror images of each other is always attractive [3]. A recently proven Earnshaw-type theorem generalises this result, demonstrating that Casimir forces between dielectric bodies of arbitrary shapes in vacuum do not support stable equilibria [4]. General duality symmetry relations relate dispersion forces between electric vs magnetic objects [5], while scaling relations constrain their dependence on body separations and sizes [6].

With experiments typically being conducted at room temperature, the impact of thermal photons on the CP potential [7] has been of particular practical interest. It has recently been studied for various non-equilibrium scenarios [8–10] where repulsive or spatially oscillating (transient) forces have been predicted [11]. On the other hand, it has also been shown that the CP potential of particles at nonretarded separations (small compared to the transition wavelengths) from a metal plate is independent of the ambient temperature, although the available thermal photon numbers at the relevant transition energies can be very large [13].

Because the geometry and temperature dependencies of dispersion forces are closely intertwined [12], one might suspect this temperature-invariance to be an artefact of the planar geometry.Quite the contrary, we will show in this Letter that this result extends to particles close to conducting bodies of arbitrary shape. The derived temperature-independent potential is governed by the electrostatic Green tensor. For geometries tractable via an image-charge method, it is simply the quantum-averaged interaction of the atomic dipole with its images as produced by the curved body surfaces.

This very simple solution to an initially complex problem of nontrivial geometry out of thermal equilibrium could greatly simplify technologically and experimentally important calculations involving such experimentally promising systems. It shows that cooling is not able to reduce or alter the nonretarded CP potential of a particle near any metal body. Two important systems for which this temperature-invariance holds in typical experimental set-ups are ground state polar molecules [11] and atoms in highly excited Rydberg states [14]. For these particles, the nonretarded regime extends to several micrometers (polar molecules) and several hundred micrometers (Rydberg atoms), respectively.

We begin by proving that the temperature-dependence of the CP force is cancelled at nonretarded separations for bodies of arbitrary shapes and give criteria for the validity of the temperature-independent result. These are illustrated by two numerical calculations for particles in nontrivial geometries.

Temperature-independence in the nonretarded regime. In this section we will show the following: Consider a particle in the vicinity of metallic bodies of arbitrary shape, such that all relevant particle–body separations are much smaller than the dominant intra-atomic transition wavelengths (nonretarded regime). Assuming further that the reflectivities of the bodies at frequencies up to the atomic transition frequencies do not differ from the static reflectivies, the CP interaction is independent of temperature.

As shown in Ref. [10], the CP potential of a particle prepared in an energy eigenstate $|n\rangle$ may be given as $U_n(\mathbf{r}) = U_n^{\mathrm{nr}}(\mathbf{r}) + U_n^{\mathrm{r}}(\mathbf{r})$ with nonresonant and resonant

contributions

$$U_n^{\mathrm{nr}}(\mathbf{r}) = -\frac{k_{\mathrm{B}}T}{\varepsilon_0} \sum_{j=0}^{\infty}{}' \mathrm{Tr}[\boldsymbol{\alpha}_n(\mathrm{i}\xi_j)\cdot\boldsymbol{\Gamma}_{\mathrm{i}\xi_j}(\mathbf{r})], \qquad (1)$$

$$U_n^{\mathrm{r}}(\mathbf{r}) = \frac{1}{\varepsilon_0} \sum_{k} n(\omega_{kn})\mathbf{d}_{nk}\cdot\mathrm{Re}\{\boldsymbol{\Gamma}_{\omega_{kn}}(\mathbf{r})\}\cdot\mathbf{d}_{kn}. \quad (2)$$

Here, $\xi_j = j2\pi k_{\mathrm{B}}T/\hbar$ are the Matsubara frequencies; $n(\omega) = [\mathrm{e}^{\hbar\omega/(k_{\mathrm{B}}T)} - 1]^{-1}$ is the mean thermal photon number of radiation of frequency ω at temperature T; $\boldsymbol{\alpha}_n$ is the atomic polarisability; and the sum in Eq. (2) runs over all states $|k\rangle$ to which there exist atomic dipole transitions with dipole moment matrix elements $\mathbf{d}_{nk} = \langle n|\hat{\mathbf{d}}|k\rangle$ and energies $\hbar\omega_{kn} = E_k - E_n$. For polar molecules [11] and Rydberg atoms [14] alike, only a few transitions to neighboring states contribute noticeably to the potential.

The tensor

$$\boldsymbol{\Gamma}_{\omega}(\mathbf{r}) \equiv \frac{\omega^2}{c^2}\mathbf{G}^{(1)}(\mathbf{r},\mathbf{r};\omega) \qquad (3)$$

is given in terms of the scattering part $\mathbf{G}^{(1)}$ of the classical Green tensor for the electromagnetic field. We shall show that this is a frequency-independent constant in the nonretarded limit. To this end it is convenient to use a Dyson equation to represent $\mathbf{G}^{(1)}$ as a perturbative expansion in $\chi/(1+\chi/3)$ where $\chi(\mathbf{r};\omega) = \varepsilon(\mathbf{r};\omega) - 1$ is the dielectric susceptibility. As shown in Ref. [15],

$$\mathbf{G}^{(1)}(\mathbf{r},\mathbf{r}';\omega) = \frac{1}{k^2}\sum_{n=0}^{\infty}\int d\mathbf{s}_1\cdots\int d\mathbf{s}_n\frac{\chi(\mathbf{s}_1;\omega)}{1+\frac{1}{3}\chi(\mathbf{s}_1;\omega)}$$

$$\times\cdots\frac{\chi(\mathbf{s}_n;\omega)}{1+\frac{1}{3}\chi(\mathbf{s}_n;\omega)}\,\mathbf{A}_{\boldsymbol{\rho}_{r1}}\cdot\mathbf{A}_{\boldsymbol{\rho}_{12}}\cdots\mathbf{A}_{\boldsymbol{\rho}_{nr'}} \qquad (4)$$

with $k = \omega/c$, $\boldsymbol{\rho}_{r1} = \mathbf{s}_1 - \mathbf{r}$, $\boldsymbol{\rho}_{12} = \mathbf{s}_2 - \mathbf{s}_1$ etc. and

$$\mathbf{A}_{\boldsymbol{\rho}} = -\frac{\mathrm{e}^{\mathrm{i}k\rho}}{4\pi\rho^3}\{[1 - \mathrm{i}k\rho - k^2\rho^2]\mathbf{I}$$

$$- [3 - 3\mathrm{i}k\rho - k^2\rho^2]\mathbf{e}_{\boldsymbol{\rho}}\mathbf{e}_{\boldsymbol{\rho}}\} \qquad (5)$$

($\mathbf{I}$: unit matrix; $\mathbf{e}_{\boldsymbol{\rho}} = \boldsymbol{\rho}/\rho$). We have assumed that all bodies are non-magnetic and isotropic, and that $\mathbf{r}$ and $\mathbf{r}'$ lie outside the dielectric bodies, where $\chi = 0$.

In the nonretarded limit $|k\tilde{z}| \ll 1$, where $\tilde{z}$ is the largest relevant particle–body distance, Eq. (5) reduces to

$$\mathbf{A}_{\boldsymbol{\rho}} \simeq -(\mathbf{I} - 3\mathbf{e}_{\boldsymbol{\rho}}\mathbf{e}_{\boldsymbol{\rho}})/(4\pi\rho^3) + \mathcal{O}(k^2)\,. \qquad (6)$$

We further assume that the perturbative parameter is frequency-independent in some low-frequency range,

$$\frac{\chi(\mathbf{r};\omega')}{1+\frac{1}{3}\chi(\mathbf{r};\omega')} \simeq \frac{\chi(\mathbf{r};0)}{1+\frac{1}{3}\chi(\mathbf{r};0)} \quad \text{for } |\omega'| \leq \omega, \qquad (7)$$

which is a good approximation for metals. With these two assumptions, the expansion (4) for the tensor (3) can be approximated by

$$\boldsymbol{\Gamma}_{\omega'}(\mathbf{r}) \simeq \boldsymbol{\Gamma}_0(\mathbf{r}) = \sum_{n=0}^{\infty}\frac{1}{(4\pi)^{n+1}}\int d\mathbf{s}_1\cdots\int d\mathbf{s}_n$$

$$\times\frac{\chi(\mathbf{s}_1;0)}{1+\frac{1}{3}\chi(\mathbf{s}_1;0)}\cdots\frac{\chi(\mathbf{s}_n;0)}{1+\frac{1}{3}\chi(\mathbf{s}_n;0)}\,\mathbf{A}_{\boldsymbol{\rho}_{r1}}\cdot\mathbf{A}_{\boldsymbol{\rho}_{12}}\cdots\mathbf{A}_{\boldsymbol{\rho}_{nr}} \quad (8)$$

for $|\omega'| \leq \omega$ together with Eq. (6), meaning $\boldsymbol{\Gamma}_{\omega}(\mathbf{r})$ is *independent* of frequency in this range.

We apply this result to the CP potentials (1) and (2) in the nonretarded regime $|\omega_{kn}\tilde{z}/c| \ll 1$. The dominant contribution to the Matsubara sum is for $\xi_j \lesssim |\omega_{kn}| \ll c/\tilde{z}$, so we can set $\boldsymbol{\Gamma}_{\mathrm{i}\xi_j} \simeq \boldsymbol{\Gamma}_0$. Performing the sum according to $\sum_{j=0}^{\infty}{}'\boldsymbol{\alpha}_n(\mathrm{i}\xi_j) = \sum_k[n(\omega_{kn})+\frac{1}{2}]\mathbf{d}_{nk}\mathbf{d}_{kn}/(k_{\mathrm{B}}T)$, we obtain

$$U_n^{\mathrm{nr}}(\mathbf{r}) \simeq -\frac{1}{\varepsilon_0}\sum_k[n(\omega_{kn})+\tfrac{1}{2}]\mathbf{d}_{nk}\cdot\boldsymbol{\Gamma}_0(\mathbf{r})\cdot\mathbf{d}_{kn}. \qquad (9)$$

Noting that $\chi(\mathbf{r};0)$ is real due to the Schwarz reflection principle, Eq. (8) shows that $\mathrm{Re}(\boldsymbol{\Gamma}_{\omega_{kn}}) \simeq \mathrm{Re}(\boldsymbol{\Gamma}_0) = \boldsymbol{\Gamma}_0$ in Eq. (2).

The total nonretarded potential hence reads

$$U_n(\mathbf{r}) \simeq -\frac{1}{2\varepsilon_0}\sum_k\mathbf{d}_{nk}\cdot\boldsymbol{\Gamma}_0(\mathbf{r})\cdot\mathbf{d}_{kn}. \qquad (10)$$

This remarkably simple expression is manifestly independent of both temperature and transition frequencies ω_{kn}. One can explicitly verify that it coincides with the zero-temperature result using $2\pi k_{\mathrm{B}}T\sum_{j=0}^{\infty}{}' \to \hbar\int_0^{\infty}d\xi$ in Eq. (1) and $n_T(\omega) \to -\Theta(-\omega_{kn})$ in Eq. (2) [Θ: unit step function]. The temperature-independent CP potential is governed by the electrostatic Green tensor $\boldsymbol{\Gamma}_0$. It is entirely due to the dipole fluctuations of the particle. For geometries where image-charge methods apply, the potential coincides with the quantum average of the interaction energy of the particle's dipole with the image dipoles inside the bodies [16]. The potential depends on the atomic internal state only via the dipole fluctuations $\sum_k\mathbf{d}_{nk}\mathbf{d}_{kn} = \langle\hat{\mathbf{d}}\hat{\mathbf{d}}\rangle_n$. There is hence no qualitative difference between forces on ground-state or excited atoms. In particular, the sign of the force is the same in both cases.

Note that the temperature-invariance holds for a particle in a chosen energy eigenstate or any temperature-independent incoherent superposition state. When the particle is in a *thermal* superposition of energy eigenstates, the CP potential acquires a weak temperature-dependence as discussed in Ref. [13]. In addition, the internal dynamics of a particle initially prepared in an energy eigenstate does of course depend on the ambient temperature.

Criteria for temperature-independence. The T-invariant form (10) of the CP potential relies on three conditions. In the following, we explicitly state these criteria and give the leading corrections due to slight violations.

(A) The relevant particle–body separations must be sufficiently nonretarded such that

$$\frac{k_{\mathrm{B}}T}{\hbar|\omega_{kn}|}\left(\frac{\tilde{z}\omega_{kn}}{c}\right)^2 \ll 1. \qquad (11)$$

Corrections due to retardation arise from corrections to Eq. (6). Provided that the approximation (7) holds, one has $\boldsymbol{\Gamma}_\omega \simeq \boldsymbol{\Gamma}_0 + \omega^2\boldsymbol{\Gamma}_0''/2$ where the primes indicate ω-derivatives and $\chi(\mathbf{r};\omega) \to \chi(\mathbf{r};0)$ is understood. We substitute this Taylor expansion into Eqs. (1) and (2). In the geometric high-temperature regime $k_{\mathrm{B}}T \gg \hbar c/\tilde{z} \gg \hbar|\omega_{kn}|$, the nonresonant potential does not contribute, because the $j = 0$ Matsubara term vanishes and all terms $j \geq 1$ are exponentially small. The retardation correction to the CP potential is hence dominated by its resonant contribution and it reads

$$\Delta U_n^{\mathrm{retard.}} = \frac{k_{\mathrm{B}}T}{2\hbar\varepsilon_0}\sum_k \omega_{kn}\mathbf{d}_{nk}\cdot\boldsymbol{\Gamma}_0''(\mathbf{r})\cdot\mathbf{d}_{kn}. \qquad (12)$$

where $\boldsymbol{\Gamma}_0''(\mathbf{r})$ is real. It can be reduced by choosing particles with larger transition wavelengths or going to smaller particle–body separations.

(B) The reflectivity of the bodies must be sufficiently frequency independent such that

$$\frac{k_{\mathrm{B}}T}{\hbar|\omega_{kn}|}\|\boldsymbol{\Gamma}_0(\mathbf{r})\|^{-1}\|\mathrm{Re}\boldsymbol{\Gamma}_{\omega_{kn}}^{\mathrm{nret}}(\mathbf{r}) - \boldsymbol{\Gamma}_0(\mathbf{r})\| \ll 1; \qquad (13)$$

Here, $\boldsymbol{\Gamma}_{\omega_{kn}}^{\mathrm{nret}}(\mathbf{r})$ is given by Eq. (4) with $\mathbf{A}$ approximated by Eq. (6), but without assuming Eq. (7). Combining Eqs. (9) and (2), the reflectivity correction for $k_{\mathrm{B}}T \gg \hbar|\omega_{kn}|$ reads

$$\Delta U_n^{\mathrm{refl.}} = \frac{k_{\mathrm{B}}T}{\hbar\varepsilon_0}\sum_k \frac{\mathbf{d}_{nk}\cdot[\mathrm{Re}\boldsymbol{\Gamma}_{\omega_{kn}}^{\mathrm{nret}}(\mathbf{r}) - \boldsymbol{\Gamma}_0(\mathbf{r})]\cdot\mathbf{d}_{kn}}{\omega_{kn}}. \qquad (14)$$

As examples of simple convex, planar and concave geometries, consider an atom at distance z from the surface of a sphere (radius $R \ll z$) [17], plate [13] or inside a spherical cavity (radius $R = z$) [18]. In these cases, one finds

$$\frac{\Delta U_n^{\mathrm{retard.}}}{U_n(T=0)} = c_{\mathrm{retard.}}\frac{k_{\mathrm{B}}T}{\hbar\omega_{kn}}\left(\frac{z\omega_{kn}}{c}\right)^2 \qquad (15)$$

with $c_{\mathrm{retard.}} = -1/3$ (sphere), $c_{\mathrm{retard.}} = 0$ (plate) and $c_{\mathrm{retard.}} = 3/5$ (cavity). The reflectivity corrections read

$$\frac{\Delta U_n^{\mathrm{refl.}}}{U_n(T=0)} = -c_{\mathrm{refl.}}\frac{k_{\mathrm{B}}T}{\hbar\omega_{kn}}\frac{z|\omega_{kn}|}{c}\mathrm{Re}\frac{i}{\sqrt{\varepsilon(\omega_{kn})}} \qquad (16)$$

with $c_{\mathrm{refl.}} = R/z$ (sphere), $c_{\mathrm{refl.}} = 6$ (plate) and $c_{\mathrm{refl.}} = 3$ (cavity). For a conductor with plasma frequency ω_P

and relaxation constant γ, the Drude model $\varepsilon(\omega) = 1 - \omega_P^2/[\omega(\omega + i\gamma)]$ leads to ($\omega, \gamma \ll \omega_P$, ω real)

$$\mathrm{Re}[i/\sqrt{\varepsilon(\omega)}] \simeq \omega_P^{-1}\left[\tfrac{1}{2}(\sqrt{\omega^2 + \gamma^2} + |\omega|)|\omega|\right]^{\frac{1}{2}}. \qquad (17)$$

The retardation and reflectivity corrections are additive to linear order. More generally, the leading correction for high temperatures can be obtained by combining Eq. (2) with the dominant $j = 0$ contribution from Eq. (1),

$$\Delta U_n(\mathbf{r}, T) = \frac{k_{\mathrm{B}}T}{\hbar\varepsilon_0}\sum_k \frac{\mathbf{d}_{nk}\cdot\Delta\boldsymbol{\Gamma}_\omega(\mathbf{r})\cdot\mathbf{d}_{kn}}{\omega_{kn}} + \mathcal{O}(T^0), \quad (18)$$

with $\Delta\boldsymbol{\Gamma}_\omega(\mathbf{r}) = \mathrm{Re}\boldsymbol{\Gamma}_{\omega_{kn}}(\mathbf{r}) - \boldsymbol{\Gamma}_0(\mathbf{r})$; it contains both retardation and reflectivity corrections.

As will be demonstrated below, the reflectivity correction is geometry-dependent. To wit, Eq. (18) distinguishes three classes of geometries where $\Delta\boldsymbol{\Gamma}_\omega \overset{\omega\to 0}{\propto} |\omega|^\eta$ with (i) $\eta > 1$, (ii) $\eta = 1$ or (iii) $\eta < 1$. In case (i), the reflectivity correction can be made arbitrarily small for sufficiently small ω, and the T-dependence can be made arbitrarily small. For case (iii), the slope of the linear correction term $\propto T$ reaches a minimum as ω decreases; it *increases* when further reducing ω because the reduction in the Green tensor difference is overcompensated by the growth of the photon number.

(C) The CP potential must be dominated by transitions for which (A) and (B) hold. If the conditions hold for a transition $n \to k$, but not for $n \to l$, then one must have

$$\left(\frac{|\mathbf{d}_{nl}|}{|\mathbf{d}_{nk}|}\right)\left(\frac{k_{\mathrm{B}}T}{\hbar|\omega_{ln}|}\right) \ll 1, \qquad (19)$$

ensuring that the linear T-corrections from retarded but suppressed atomic transitions do not contribute. Care must be taken in the case where a transition resonates with a cavity of quality-factor $\mathcal{Q}$. Then we even require that $\mathcal{Q}(|\mathbf{d}_{nl}|/|\mathbf{d}_{nk}|)(k_{\mathrm{B}}T/\hbar|\omega_{ln}|) \ll 1$.

Numerical examples. We now present two illustrative examples of non-planar geometries with curvature radii on the order of the particle–body separation. For simplicity, we consider isotropic particles for which

$$\mathbf{d}_{nk}\cdot\boldsymbol{\Gamma}_\omega(\mathbf{r})\cdot\mathbf{d}_{kn} = \tfrac{1}{3}|\mathbf{d}_{nk}|^2\mathrm{tr}\boldsymbol{\Gamma}_\omega(\mathbf{r}). \qquad (20)$$

Consider first the case of a particle at distance r from the center of a gold sphere of radius R. The Green tensor for this geometry is given in Ref. [20]. In the nonretarded limit $kr, kR \ll 1$ and for a perfectly conducting sphere $|\varepsilon| \to \infty$, Eqs. (10) and (20) lead to

$$U_n(\mathbf{r}) \simeq -\frac{R^3(6r^4 - 3r^2R^2 + R^4)}{24\pi\varepsilon_0 r^4(r^2 - R^2)^3}\sum_k |\mathbf{d}_{nk}|^2. \qquad (21)$$

This T-independent potential is in agreement with the zero-temperature result found in Ref. [21]. In Fig. 1, we
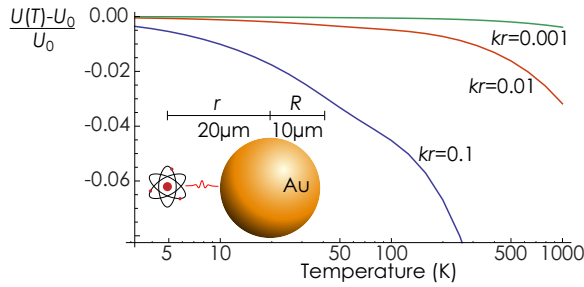
FIG. 1: Comparison of the exact thermal CP potential $U(T)$ of a ground-state two-level particle (transition energy $\hbar ck$) outside a gold sphere with the T-independent result U_0 from Eq. (10).

FIG. 2: Same as Fig. 1, but for a particle inside a gold cylindrical cavity.

demonstrate the accuracy of Eq. (21) over a large range of temperatures for particles of different transition frequencies outside a gold sphere. The sphere is described by the Drude model with $\omega_P = 9\text{eV}$ and $\gamma = 35\text{meV}$. As Fig. 1 shows, the correction due to finite temperature is about 1% at room temperature when $kr = 0.01$ and decreases rapidly as $kr \to 0$, thus exemplifying case (i) with $\eta = 3/2$. The temperature correction due to retardation (A) is the dominant correction, and the reflectivity correction (B) is considerably smaller.

Consider next a particle inside a cylindrical cavity of radius R whose Green tensor can be found in Ref. [19]. In the nonretarded limit and for a perfectly conducting sphere, it leads to the potential

$$U_n(\mathbf{r}) \simeq - \int_0^\infty \frac{dq}{6\pi^2\varepsilon_0} \sum_{m=0}^\infty {}' \frac{K_m(qR)}{I_m(qR)} \left\{ \left[(m/\rho)^2 + q^2 \right] \right.$$
$$\left. \times I_m^2(q\rho) + q^2 I_m'^2(q\rho) \right\} \sum_k |\mathbf{d}_{nk}|^2 \qquad (22)$$

(K_m, I_m: modified Bessel functions). Its reliability is illustrated in Fig. 2. In contrast to the previous example, the reflectivity correction (B) dominates; it may be shown that except very close to the walls one has case (iii) with $\eta \approx 1/2$. Hence as the particle's transition frequency is decreased, the temperature corrections reach a minimum. Note that this problem does not arise when suppressing these corrections by reducing the spatial dimensions of the setup. The temperature dependence is nonetheless quite modest; for our parameters the linear T-correction is at the 5% level at room temperature for $kR \sim 0.01 - 0.05$.

Conclusions. We have shown that the CP potential of a particle prepared in an energy eigenstate at nonretarded distance from a conducting body may be approximated by an expression which is independent of both temperature and particle transition frequency, regardless of the geometry of the body. In this regime, the interaction is governed by the transition dipole matrix elements of the particle and the electrostatic Green tensor. There is no qualitative differ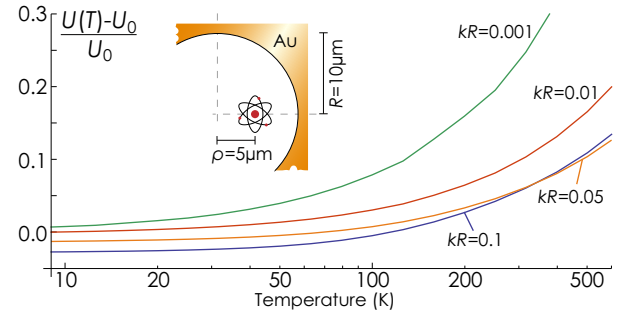ence between the potential acting on atoms in ground or excited states. For instance, this implies that repulsive CP forces that have been predicted for ground-state atoms in certain geometries [22] may be more easily verified using excited particles in the same geometry.

While the fact that dispersion interactions are of electrostatic nature in the nonretarded regime is well known, the remarkable cancellation of temperature dependent contributions only occur once both nonresonant and resonant contributions are fully accounted for. Each of these force components, which arise from physically distinct phenomena, individually depend strongly on temperature and transition frequency.

The two numerical examples illustrate the reliability of the temperature-independent potential. For a particle next to a metallic sphere, the temperature dependence vanishes for ever smaller values of the retardation parameter $r\omega/c$, whereas for a particle in a cylindrical cavity, the T-dependent corrections reach a minimum beyond which they cannot be further reduced. The latter effect seems to be a peculiarity of the cylindrical geometry.

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